Assessment of significant wave height correlation distances in the North Sea and North East Atlantic using a mesoscale wave hindcast

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Introduction

Applications of observed data for model verification and data assimilation require an understanding of spatial scales at which the conditions are observed and model verus observation errors are correlated. For example, in data assimilation it is necessary to estimate the background error covariance matrix. For verification the requirement is twofold. In the first instance an idealised verification will be carried out on samples of statistically independent events, i.e. the events and error samples can be demonstrated to be correlated only at the level of random chance.

The second requirement is to understand if observations from different locations are sufficiently correlated to accept an assumption that the same condition has been observed. In verification studies it is often assumed that at an observation represents the 'truth' and any deviation from this value is the model 'error'. This approach assumes that any errors in the observations are small and can be discounted. Triple collocation studies e.g. Janssen et al., (2007) have shown that observation errors are not always insignificant and should be accounted for. It has been shown in the literature that significant wave height (Hs) measurements from satellite altimeters typically have an error of about 5 - 10 % (Cotton and Carter 1994, Janssen et al., 2007). On a global scale Janssen et al. (2007) demonstrated an error from in-situ observations of approximately 8%, whilst Durrant and Greenslade (2009) showed significant discrepancies between the in-situ observations provided by US and Canadian observing networks in the northeast Pacific.

The method underpinning triple collocation is based on deriving error estimates for 3 or more independent observing/prediction systems, based on their description of the same sample of events; in most triple collocation studies of wave heights these are taken from in-situ measurements, satellite altimeters and a wave model. Whilst a model and in-situ measurement can be collocated exactly, precise spacio-temporal collocation between am in-situ measurement and satellite sounding would be sufficiently rare to render the method unfeasible. As a result the three data sources are assumed to be collocated when a satellite measurement is made within a predefined time and distance from the in-situ observation. An assessment of high correlation between locations in the wave field provides a quantitative method of determining these criteria.

In this study the aim is to establish spatial correlation scales that allow a determination of distances at which high correlation in the wave field can be assumed, in order to optimise a triple collocation method, and enable an estimate of the distances at which correlation is low enough to assume that model verus observation errors will be independently sampled. The work follows a technique used for data assimilation in order to estimate the background error covariance matrix. Predicting the structure of this field is a significant challenge and a number of methods have been developed in the literature (e.g. Lonnberg and Hollingsworth, 1986, Greenslade and Young, 2005). The method used in this case follows the Canadian Quick Covariance technique (Polavarapu, 2005) and is discussed further in the next section. A further difference from previous wave studies is the application to regional sea areas, including a basin with significant areas of shallow water (the North Sea). This has necessitated the use of a relatively high resolution background field. Subsequent sections provide a summary of the study results and conclusions relevant to regional triple collocation and verification testing to be undertaken within the European Union sponsored MyWave project[1].

[1] http://mywave.eu/en/

Methodology

Calculating spatial covariance fields

To calculate a covariance field between any one location and other points within a domain it is necessary to use model data to provide an estimate, as observations are sparse. To estimate the degree of correlation between a single location and all other grid cells in a model domain, the covariance field between the grid cell at that location and at all other grid cells is calculated using equation 1.

$$Co = E\left[\left(x - E[x]\right)_i \left(x - E[x]\right)_j\right] \quad (1)$$

where E[*x*] is the mean, and *i* in this case is the model grid cell at the specified location and *j* represents all other grid points in the domain.

Calculating a covariance field to approximate background error

Estimating the background error covariance matrix is recognised to be one of the most difficult tasks when implementing data assimilation in numerical models. There is limited research as to the best methods for this in the case of wave models. When the model domain includes coastal areas, there is the added difficulty of accurately representing the covariance in areas where the wave conditions may vary on relatively short spatial scales. To date three main approaches have been used: Lonnberg andHollingsworth (1986), ensemble methods (e.g. Greenslade and Young, 2005) and standard statistical correlation functions (e.g. Gaspari and Cohn 1999).

Lonnberg and Holllingsworth's (1986) method uses the difference between the model output and observations. This approach assumes that the background errors and observational errors are not correlated. The background error covariance matrix is estimated from the mean ensemble difference between the observations and the model. This approach is not generally successful where observations are sparse.

An alternative approach is to use an ensemble method; this uses the relationship between a number of model simulations. This approach has the advantage of not needing large numbers of observations; however it does require large numbers of model simulations which are computationally expensive. An example of such a technique is the NMC (U.S. National Meteorological Center, now the National Centers for Environmental

Prediction, NCEP) method as used by Greenslade and Young (2005). This method uses an ensemble of forecast differences, by examining the differences between forecasts of different ranges (lead times)that are valid at the same time.

A cheaper approach to calculating the background error covariance is the 'Canadian Quick Covariance (CQC) method, which was first implemented for data assimilation by Polavarapu et al. (2005) in the Canadian Middle Atmosphere Model. In this scheme the background error is estimated from the covariance of the hourly difference field between model states. The CQC method has subsequently been implemented for a data assimilation scheme in a wave model by Waters (2010). The scheme implemented by Polavarapu et al. (2005) used the difference fields between the 6 hourly model outputs; it was thought that a 6 hourly background error would be comparable with the spatial structure of the 6 hourly difference field (equation 2).

$$Co = E\left[\left((x_{t+6} - x) - (E[x_{t+6}] - E[x])\right)_i \left((x_{t+6} - x) - (E[x_{t+6}] - E[x])\right)_j\right] \quad (2)$$

This approach has been tested successfully by other authors (e.g. Waters, 2010). The method doesn't require a model with data assimilation included and so can be applied to any pre-existing hindcast.

Study Method

For the regional study described in this report the CQC method has been applied to a high resolution model in order to resolve spatial structure. Model data for the study were taken from a readily available 10-year hindcast of waves in European waters. The hindcast was based on an 8km configuration of WAVEWATCH III™ (Tolman, 2009) as the wave model (see figure 1 for domain) which was driven by winds from a 12km instance of the Met Office Unified Model (Davies et al., 2005). Wave boundary conditions were provided by a 50km global WAVEWATCH III configuration driven from ERA-Interim.

In particular the study focuses on spatial covariance between a number of data points corresponding to in-situ locations and the wider model domain (figure 2). The model background error covariance estimates were calculated over 3 monthly intervals for the year 2009. This was to determine whether there were any seasonal variations in the structure of the covariance field due to seasonal variability in storm systems that drive

the wave climate in the northeast Atlantic and northwest European shelf seas. An hourly difference field was used to approximate the hourly background error because this gives a better temporal resolution of the forcing fields and the variability in the model is greater in coastal regions.

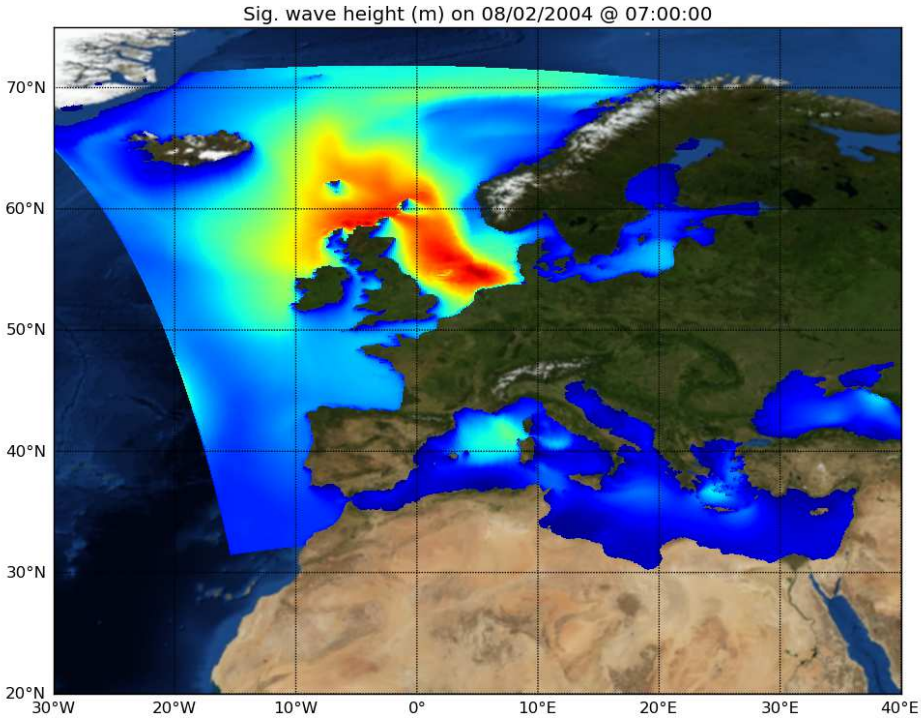


Figure 1. Domain of 8km WAVEWATCH III™ European Model.

Fitting a Correlation Function

Correlation fields (calculated based on the covariance in equation 2) were calculated for points on the model grid shown in figure 2. The correlation field was then divided into 4 segments; north, south, east and west, based on the bearing of each point in the correlation field from the grid point. For example the north segment used all grid point values on a bearing between 315º and 45º, east values consisted of grid points that lay on a bearing of 45º to 135º and so forth for each 90º segment. Correlation was plotted for each segment as a function of distance.

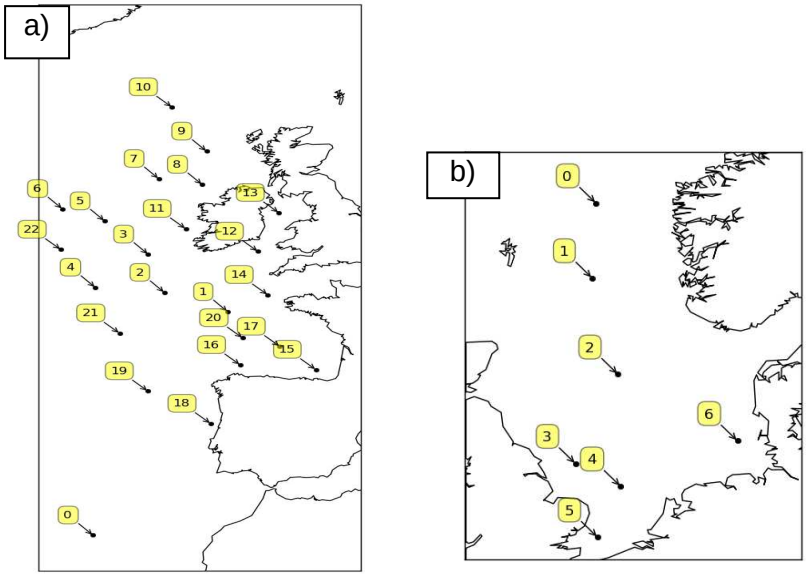


Figure 2 a & b Points selected for correlation study in
a) North East Atlantic Margin and b) North Sea.

In order to determine correlation length-scales for the background error it is necessary to fit a function to the correlation field. There are a number of approaches available, including a simple one dimensional function, where it is assumed that the correlation is isotropic, and two dimensional functions that allow for anisotropic correlation. The assumption that the background error is isotropic is unlikely to be correct in the case of a wave model domain.

Standard statistical correlation functions produce a positive, symmetrical and smooth background error covariance. These are usually calculated by fitting an analytical function (e.g. Gaussian or SOAR function) to the data. The correlation function may also be calculated empirically. Fitting a correlation function involves determining the function's length-scale. A classical definition of the differential length-scale is given by Daley (1991). The length-scale diagnosis describes the curvature of the correlation functions near the origin. The smaller the length-scale, the faster the correlation decreases with distance (Pannekouche et al., 2008). In this study the distance to a

particular correlation level is the required output and was estimated from the fitted correlation function values. Correlation was separated into four directional sectors (north, south, east, west) and a simple one dimensional correlation function was then fitted to each sector. This approach assumes that the correlation is isotropic within each sector.

From the literature a number of different forms of the correlation function were tested on the data. The correlation function takes the general form (equation 3):

$$P_{kj} = \left(1 + \frac{\left[\left|x_k - x_j\right|\right]^a}{L}\right) \exp\left[-c\left(\frac{\left|x_k - x_j\right|^b}{L}\right)\right] \quad (3)$$

where L is the decorrelation length scale and $\left|x_k - x_j\right|$ is the distance between points *j* and *k*. The values of *a, b, c* and *L* have taken various values in the literature and are usually fitted to the data using a least squares method.

A Gaussian and SOAR function were compared to the data to determine which form of correlation function would provide the best fit. A Gaussian form of the correlation function did not fit the correlation data near the origin and was therefore unsuitable. A reasonable fit was achieved using the form of the correlation function used by Voorips (1997) (equation 4).

$$P_{kj} = \exp\left(-\frac{\left|x_k - x_j\right|^a}{L}\right) \quad (4)$$

The values used by Voorips (1997) for L and a, were 200(km) and 3/2 respectively. This form of the correlation function was able to fit the data near the origin, but did not provide a good fit at greater distances.

It was found that a good general fit to the data at all distances could be achieved using a SOAR function with two length scales. This is shown in equation 5:

$$P_{kj} = a\left(1 + \frac{\left|x_k - x_j\right|}{L_1}\right)\exp\left(-\frac{\left|x_k - x_j\right|}{L_1}\right) + b\left(1 + \frac{\left|x_k - x_j\right|}{L_2}\right)\exp\left(-\frac{\left|x_k - x_j\right|}{L_2}\right) \quad (5)$$

A least squares method was used to fit the function to the data. The value of b was constrained to (1-a) in order for the fitting method to return suitable values, where P can

take a maximum value of 1.0. The lengthscale values (*L*) were also constrained to only take positive values. In some cases the domain boundary or presence of the coastline did not allow the correlation to reduce to zero, which resulted in large lengthscales being estimated at low correlation values at these locations. However this did not affect the fitting of the function closer to the origin at correlation levels above 0.5. In a few cases an optimum solution was not found and it was not possible to fit the function to the data. This is likely to be due to the scatter in the data in some cases where there was large variability in the correlation values with distance. In the case where a satisfactory solution was not obtained with the SOAR function, it was possible to fit an alternative form of the correlation function to the data. However it was decided that it would be more consistent to exclude these data rather than to use an alternative correlation function for these cases

Results

Structure of the background error covariance matrix and distances of highly correlated values

The CQC analysis and lengthscale fit were applied for locations corresponding to in-situ sites in two predefined regions. The North Sea domain covers an area between 51° N and 63°N, has a climate dominated by locally driven wind-seas and includes areas of relatively shallow water (0-40m) in its southern sector. The North East Atlantic Margin (NEAM) area falls between lat-lon 30°N, 20°W and lat-lon 0°W, 65°N and has a climate comprised of seas and swells generated further west in the North Atlantic.

The background error covariance field is highly variable for different points in both study areas. In this study highly correlated background errors have been defined as those with a correlation greater than 0.8. The average decorrelation distances in the NEAM for each direction and season range from 70 – 159km (see Table 1). At points located close to the coast the decorrelation distance decreases more rapidly in the direction of the coastal zone due to changes in wave conditions in shallow water, obstruction due to islands and the shape of the coastline.

Figure 3 illustrates how correlation can drop off rapidly in the coastal zone, in this case either side of headlands. The decorrelation distance of the background error is much greater to the west and the deep ocean. It should also be noted that proximity to the coast can result in little or no decorrelation between the point of interest and the coast. An example would be the easterly direction in figure 3.

Decorrelation distances are shorter overall in the North Sea, on average 40– 50 km less than the NEAM. The range to the 0.8 correlation level is 35km – 115km for the four different directions and seasons (Table 2). Figure 4 is an example of a point in the southern North Sea close to the coast.

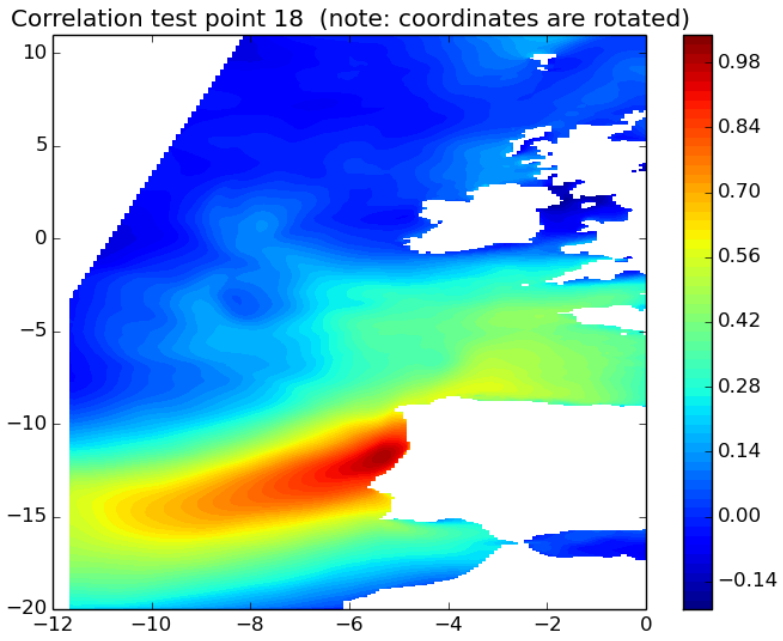


Figure 3 Correlation field for NEAM point positioned near the coast (Winter)

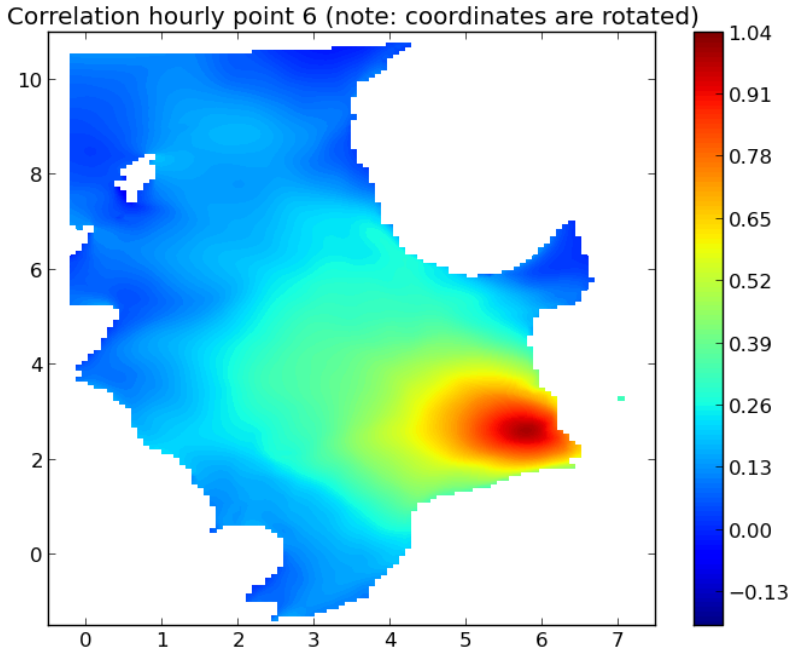


Figure 4 Correlation point in the south North Sea near the coast (Winter)

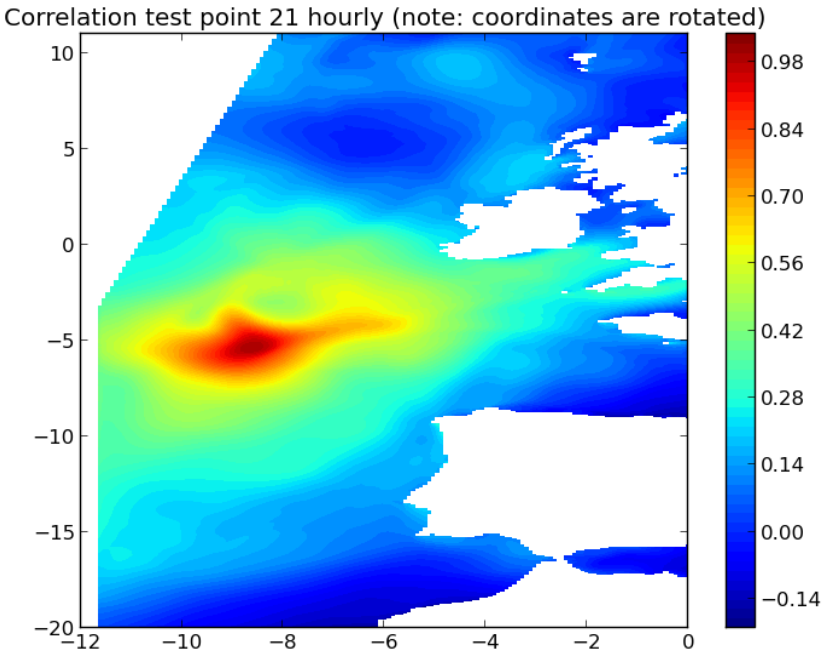


Figure 5 Correlation field for NEAM point positioned in the deep ocean (Winter)

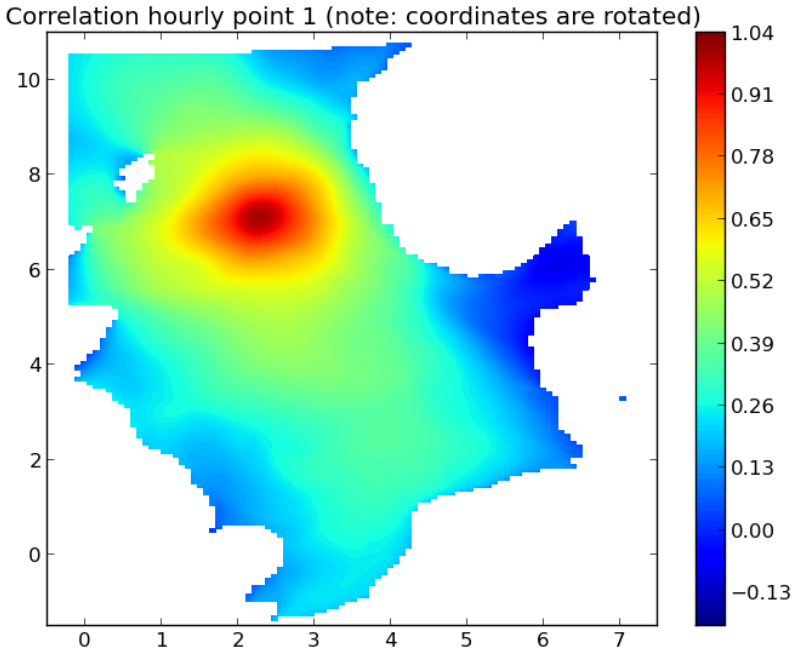


Figure 6 Correlation point in the North Sea positioned in the north (Winter)

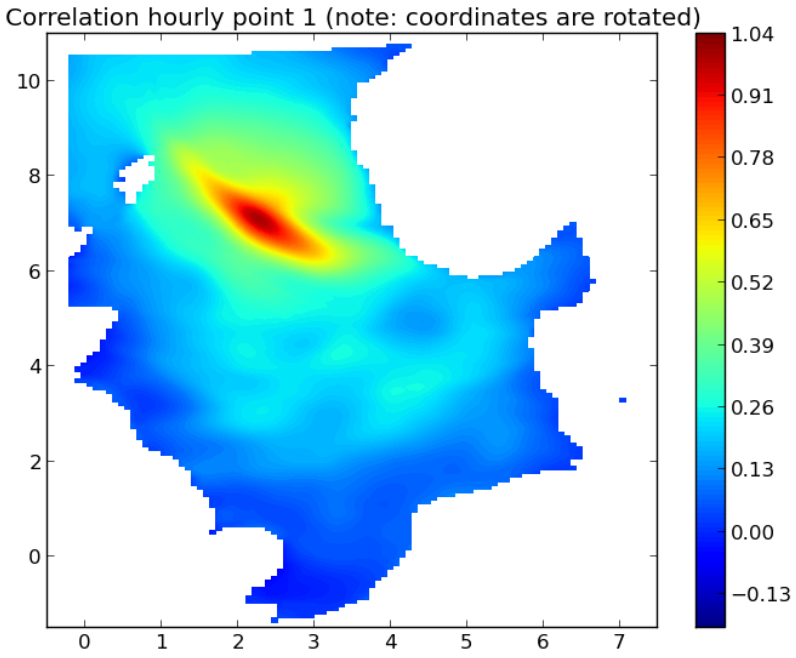


Figure 7 Correlation point in the North Sea positioned in the north (summer)

Directional Variations

In the NEAM the longest decorrelation distances are seen in the East and West directions with a range of 106 – 159 km east and west. In the north and south sectors the decorrelation distances range from 70 – 115km, for a correlation value of 0.8 (Table 1). Figure 5 shows how the rate of decorrelation varies with direction, the correlation of background error generally decreases more rapidly to the north and south. Greenslade and Young (2005) found that in the North Atlantic the longest axis for decorrelation were aligned in a northeast direction. It was suggested that this indicated that the structure of the background errors were mainly due to errors in the generation of swell systems within the model, propagating from the northwest.

In this study the longest axis for decorrelation in the NEAM was in an easterly direction (figure 5), the longest decorrelation distance is actually aligned between an easterly and north easterly direction, suggesting a similar result to Greenslade and Young (2005). Swell systems observed in open waters off the coast of southwest UK, France and Spain, predominantly propagate from the northwest in the North Atlantic, suggesting that

in this case the longest axis is also perpendicular to the direction of the swell. Storm systems also track from further south moving in a more easterly direction but this is less common.

In the North Sea longer decorrelation distances also occur, on average, in the east and west sectors, with a range from 73 – 115km. The north and south sectors ranged on average for each season from 35 – 61km for a correlation value of 0.8 (Table 2). The longest decorrelation distances in the summer were observed from north west to south east, in the winter the decorellation distance was similar in all directions. Storms frequently track from northwest Iceland towards the north of Scotland. Swell systems from these storms may propagate from north to south in the North Sea. It is also possible that local storms may be more significant in determining the structure of the background error correlation field in the North Sea. The longest fetch is from north to south and this may affect the alignment of the background error correlation field.

Seasonal variations

Figures 6 and 7 show the background correlation for the same point in the north of the North Sea in winter and summer respectively. Significantly shorter decorrelation distances were estimated for the summer. It is also of note in this example that the summer correlation field is asymmetrical, this suggests that seasonal changes in the predominant fetch (as well as overall wave heights) will effect the structure of the background error correlation field.

In the NEAM the longest correlation distances occur in the Spring and Autumn, the shortest are calculated for the Summer. The differences between the seasons are significant with correlation distances 30-60km less on average for each directional sector in the summer for the 0.8 correlation level and differences in distance of over 100km for the 0.5 correlation level (Table 3).

In the North Sea the longest decorrelation distances were calculated for the winter, these are about 30km greater on average for each directional sector at the 0.8 correlation level than the distances calculated for the summer (Table 2). For the 0.5 correlation level seasonal differences between winter and summer are in the range of 100km on average for each directional sector (Table 4).

Table 1 NEAM 0.8 Correlation distance (km)

	Spring	Summer	Autumn	Winter
North	**104.8**	**70.2**	**100.9**	**88.3**
South	**112.7**	**71.6**	**115.2**	**84.8**
East	**152.3**	**106**	**149.5**	**116.7**
West	**156.4**	**108.3**	**159.8**	**115.0**

Table 2 North Sea 0.8 Correlation distance (km)

	Spring	Summer	Autumn	Winter
North	**49.2**	**36.4**	**45.0**	**60.7**
South	**44.3**	**35.0**	**43.7**	**52.8**
East	**85.0**	**75.0**	**91.4**	**94.0**
West	**81.4**	**72.9**	**92.8**	**115.0**

Table 3 NEAM 0.5 correlation distance (km)

	Spring	Summer	Autumn	Winter
North	**230.7**	**160.2**	**234.1**	**208.9**
South	**281.6**	**180.0**	**286.3**	**244.8**
East	**336.7**	**274.5**	**356.4**	**327.6**
West	**380.5**	**256.0**	**408.7**	**353.7**

Table 4 North Sea 0.5 Correlation distance (km)

	Spring	Summer	Autumn	Winter
North	**145**	**107.1**	**116.4**	**180.0**
South	**120.7**	**85.7**	**250.7**	**177.1**
East	**194.3**	**117.5**	**221.4**	**222.0**
West	**181.4**	**172.9**	**211.4**	**279.2**

Observation and Model Comparisons

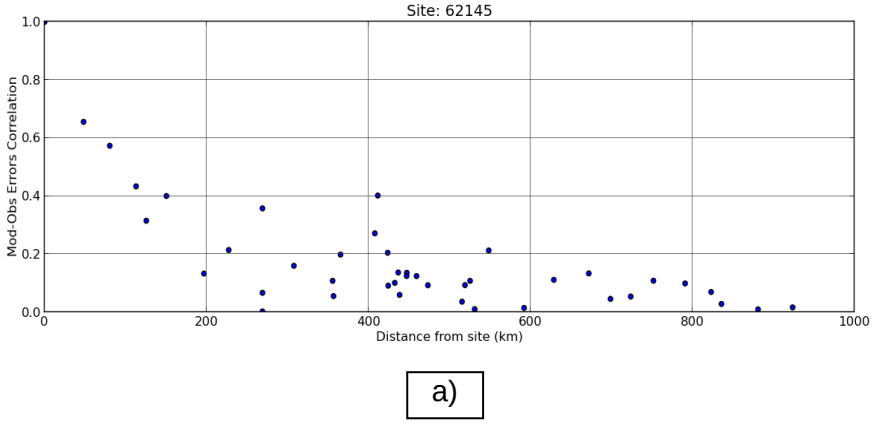
The background error correlation estimated in this study using the Canadian Quick Covariance (CQC) method relies on the assumption that the hourly difference field is an approximation of the structure of the background error field. A method of testing this assumption is to compare the CQC data with correlation scales for actual model verus observation errors. The Lonnberg and Hollingsworth's (1986) approach was used to determine the correlation of the background errors. In the regions assessed in-situ observation coverage is sufficient to allow such a comparison to take place, albeit for a limited sample.

North Sea Points

Figures 8 to 12 show a comparison of the model-observation error correlation with the results of the CQC method for the north and west directions. There is a large degree of variability seen in all the results, both in the model – observation comparisons and the data generated by the CQC method. This is to be expected given the directional variability in the CQC data that have already been described above. Figure 8 is an example taken from the North Sea. In figure 8a the first data point for the error correlation is at 0.6. There is a lack of data to describe the distance for higher correlation levels at this point. This first point shows the 0.6 correlation level to be at approximately 50km. The second data point is also at a correlation of 0.6 and is at about 100km. The CQC method indicates a correlation level of 0.6 at about 200km in both the northerly and westerly directions. The slightly longer distance is seen in the westerly direction in figure 8c.

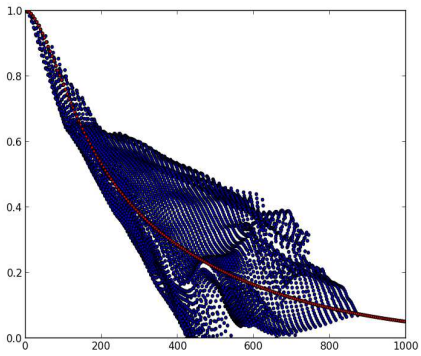
Figure 9 is another point taken in the north of the North Sea, where there are more observational data available for high correlation distances. The observation-model comparisons show the 0.8 correlation level to be at less than 50km. The 0.5 correlation level is at 100 to 200km, this indicates a large variability in the lower correlation values with distance. The background error correlation estimated from the CQC method for the north sector calculates a distance of approximately 50km for the 0.8 correlation level and 200km for the 0.5 level. In the west sector the correlation distances are longer, with 0.8 at over 100km and 0.5 at 250- 300km. It should be noted that for this study all examples

from the CQC have been taken from the winter when correlation distances are generally longer.



a)

b)



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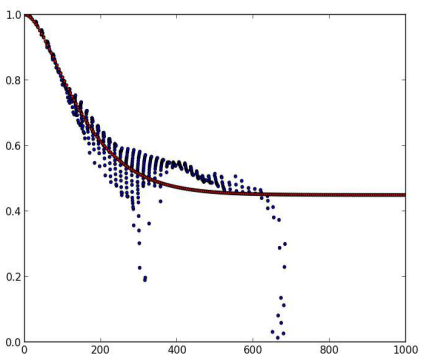
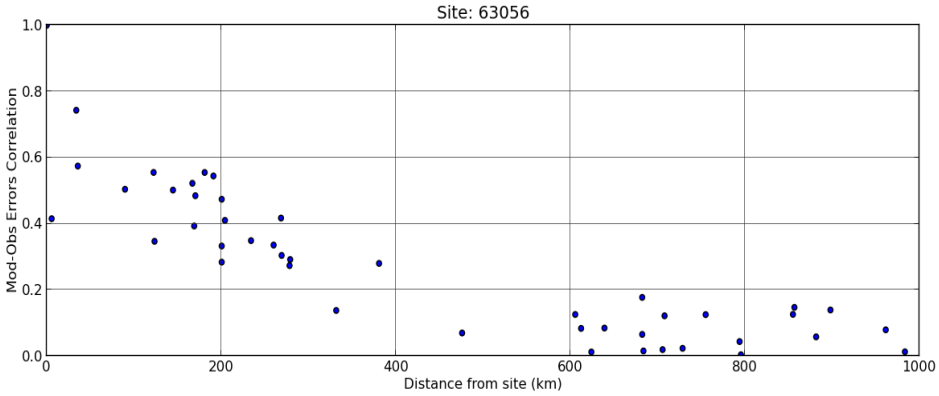


Figure 8 Correlation against distance at buoy site 62145 a) model-observation error correlation b) CQC background error correlation in north sector c) CQC background error correlation in west sector.



a)

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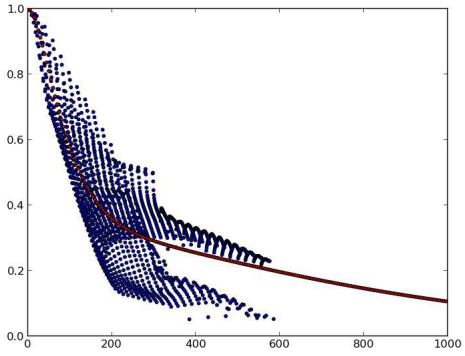

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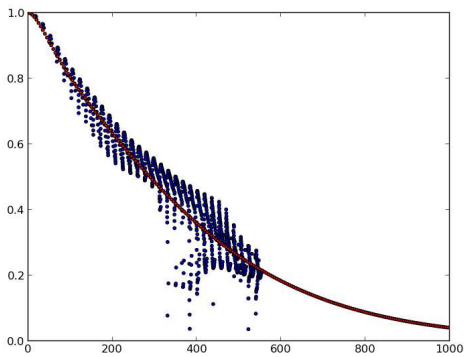

Figure 9 Correlation against distance at buoy site 63056 a) model-observation error correlation b) CQC background error correlation in north sector c) CQC background error correlation in west sector.

NEAM Points

Figure 10 is a NEAM point, the model-observation error correlation (figure 10a) results show an approximately 0.5 correlation at 300km. The CQC data show the 0.5 correlation level to be 250km in the north sector and 350km in the west sector figure 10b and c.

In figure 11a the 0.5 correlation for the model-observation error is shown at about 200km. The CQC method estimates this to be about 250km in the north sector figure 10b and 350km in the west sector (figure 11c). The westerly direction is close to the UK at this point, therefore this is likely to affect the results. The easterly direction shows the 0.5 correlation level to be about 380km. The 0.4 error correlation level is shown as about 400km in the model-observation data (figure 10a), this is also the case in the CQC data for the north sector (11b).

In figure 12a the first data point for the model-observation error correlation is at the 0.4 correlation level at a distance of about 200km. There is again a lack of data for the distances of the higher correlation values. Figure 12b shows the data for the CQC method in the north section, the 0.4 correlation is estimated to be approximately 200km. There is a lack of data at a distance of over 200km in the westerly direction, the easterly direction shows that the 0.4 correlation level is reached at about 270km.

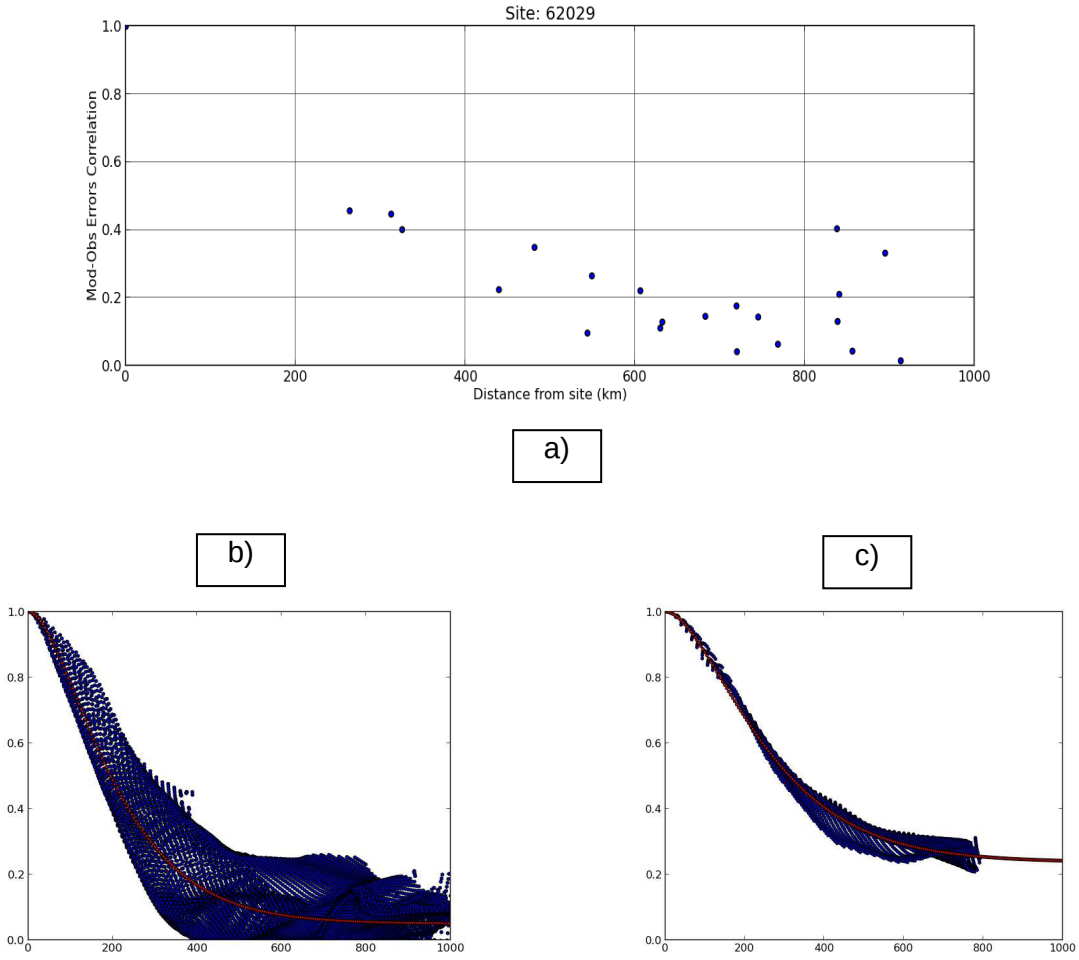

Figure 10 Correlation against distance at buoy site 62029 a) model-observation error correlation b) CQC background error correlation in north sector c) CQC background error correlation in west sector.

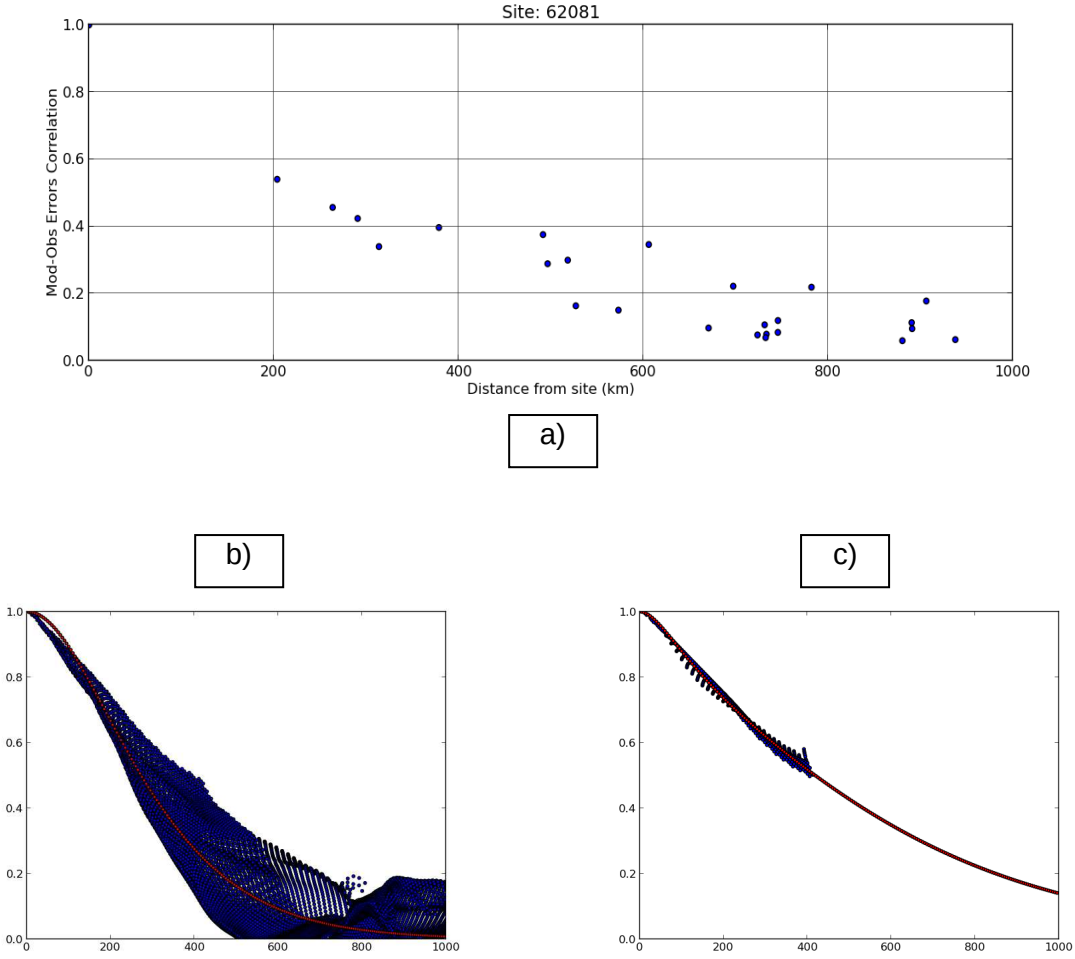

Figure 11 Correlation against distance at buoy site 62081 a) model-observation error correlation b) CQC background error correlation in north sector c) CQC background error correlation in west sector.

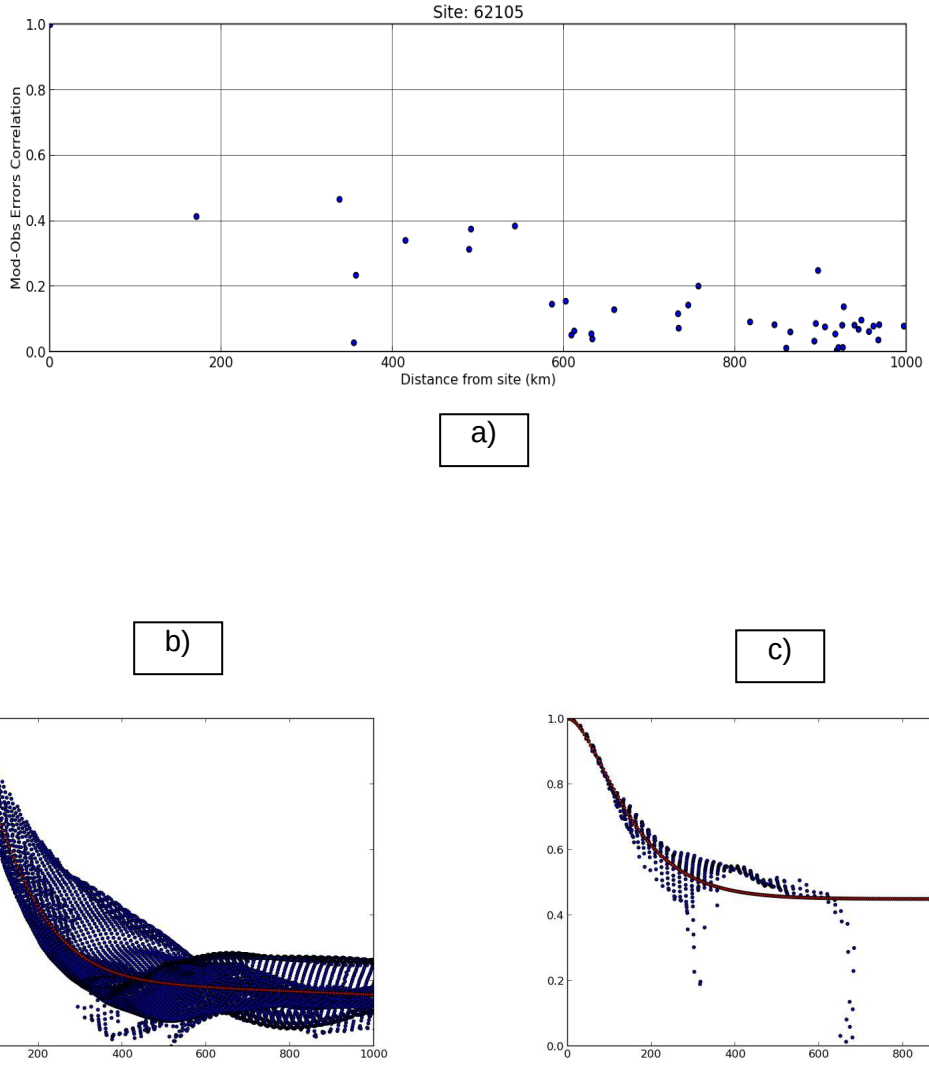

Figure 12 Correlation against distance at buoy site 62105 a) model-observation error correlation b) CQC background error correlation in north sector c) CQC background error correlation in west sector.

Conclusions

This study has applied the Canadian Quick Covariance technique, for deriving background error covariance, to a mesoscale wave hindcast in order to assess correlation distances in two regional European sea areas. It has been possible from the results to establish the scales where background error is spatially correlated in the NEAM and North Sea. Comparisons of the CQC method results with error correlations determined for actual model-observation differences from regional in-situ networks show that the CQC estimate of spatial correlation is a reasonably close representation. The spatial correlation of background errors generally leads to a slightly longer decorrelation distance using the CQC method.

For this study highly correlated background errors have been defined as distances where the correlation is greater than 0.8. In a triple collocation study this level of correlation represents where background error values are sufficiently correlated for there to be confidence that measurements are of the same wave conditions and can be used to select the maximum distance at which different measurement sources can be compared. In both the NEAM and North Sea the shortest distances for this correlation level were found to be in the north- south directions. In these directions a distance of 115km in the NEAM and 60km in the North Sea would be the maximum distances at which two different measurements could be collocated. In the east – west direction of the North Sea a spatial distance between different measurement sources of 115km would be the maximum distance, in the NEAM this distance could be extended to 160km.

A low correlation criterion (<0.5) is used in this study to define distances at which conditions and model – observation errors are sampled independently. In areas such as the North Sea, applying this criterion in verification studies may be important to ensure that sampling is not biased due to the presence of closely located in-situ sites. From the results of this study the required distances between sample points in the North Sea would be about 250km in the north-south direction and 279km east – west. In the NEAM these distances are 286km north – south and 409km east – west. The distances to the 0.5 correlation level are variable; the values given here are the maximum average distance for each season and direction.

In the NEAM the background correlation structure differs in coastal areas from deep-sea locations. There is also a large seasonal variation in both domains. It may be useful to consider the coastal and deep-sea location distances separately in the NEAM to ensure that data samples are not reduced unnecessarily. In practice, in coastal areas measurement points may be closer together than the distances given here, but still experience different wave conditions. Overall there was significant directional dependency in the structure of the background error correlation. Generally spatial correlation distances were found to be greater in the west to east directions in both domains. This is believed to be due to the predominant direction of travel for swell fields and storm fronts, where the wave energy will be distributed relatively evenly over the frontal region. Similar results have been found in previous studies e.g. Greenslade and Young (2005).

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